Articulatory-Acoustic Relations: From (very) basic principles to simulation
Paris Diderot
Lecture 1

Khalil Iskarous
University of Southern California

November 24, 2015
Overview

Forward Problem: Given a vocal tract configuration, what's the resulting spectrum.

Inverse Problem: Given a spectrum, what types of vocal tract configurations could have generated this spectrum.

Signal processing: Given a signal output from the vocal tract, how do we extract as much information from that signal.
Overview

- **Forward Problem**: Given a vocal tract configuration, what’s the resulting spectrum.
Overview

- **Forward Problem**: Given a vocal tract configuration, what’s the resulting spectrum.
- **Inverse Problem**: Given a spectrum, what types of vocal tract configurations could have generated this spectrum.
Overview

- **Forward Problem**: Given a vocal tract configuration, what’s the resulting spectrum.
- **Inverse Problem**: Given a spectrum, what types of vocal tract configurations could have generated this spectrum.
- **Signal processing**: Given a signal output from the vocal tract, how do we extract as much information from that signal.
A physical feel for vibration of the air in F1 for schwa. Why does the air vibrate in this way? It’s a bit easier to think about strings.
First mode of vibration of a string
Second mode of vibration of a string
Third mode of vibration of a string
The answer has to do with a very useful equation that’s at the center of a great deal of mathematical physics: the wave equation.

What you need to understand to appreciate the wave equation:

1. $u$: Position of a particle
2. $\frac{du}{dt}$ or the Velocity of a particle: the rate of change of position with respect to time.
3. $\frac{d^2u}{dt^2}$ or the Acceleration of a particle: the rate of the rate of change of position with respect to time.
4. $\frac{du}{dx}$ Spatial gradient/slope: the position of a particle with respect to some surrounding particle.
5. $\frac{d^2u}{dx^2}$ Spatial curvature: the change of the slope with respect to space.
Wave Equation

- The answer has to do with a very useful equation that’s at the center of a great deal of mathematical physics: the wave equation.
The answer has to do with a very useful equation that's at the center of a great deal of mathematical physics: the wave equation.

What you need to understand to appreciate the wave equation:
The answer has to do with a very useful equation that’s at the center of a great deal of mathematical physics: the wave equation.

What you need to understand to appreciate the wave equation:

1. $u$: Position of a particle
The answer has to do with a very useful equation that’s at the center of a great deal of mathematical physics: the wave equation.

What you need to understand to appreciate the wave equation:

1. \( u \): Position of a particle
2. \( \frac{du}{dt} \) or the Velocity of a particle: the rate of change of position with respect to time.
The answer has to do with a very useful equation that’s at the center of a great deal of mathematical physics: the wave equation.

What you need to understand to appreciate the wave equation:

1. $u$: Position of a particle
2. $\frac{du}{dt}$ or the Velocity of a particle: the rate of change of position with respect to time.
3. $\frac{d^2 u}{dt^2}$ or the Acceleration of a particle: the rate of the rate of change of position with respect to time.
The answer has to do with a very useful equation that’s at the center of a great deal of mathematical physics: the wave equation.

What you need to understand to appreciate the wave equation:

1. \( u \): Position of a particle
2. \( \frac{du}{dt} \) or the Velocity of a particle: the rate of change of position with respect to time.
3. \( \frac{d^2u}{dt^2} \) or the Acceleration of a particle: the rate of the rate of change of position with respect to time.
4. \( \frac{du}{dx} \) Spatial gradient/slope: the position of a particle with respect to some surrounding particle.
The answer has to do with a very useful equation that’s at the center of a great deal of mathematical physics: the wave equation.

What you need to understand to appreciate the wave equation:

1. $u$: Position of a particle
2. \( \frac{du}{dt} \) or the Velocity of a particle: the rate of change of position with respect to time.
3. \( \frac{d^2 u}{dt^2} \) or the Acceleration of a particle: the rate of the rate of change of position with respect to time.
4. $\frac{du}{dx}$ Spatial gradient/slope: the position of a particle with respect to some surrounding particle.
5. $\frac{d^2 u}{dx^2}$ Spatial curvature: the change of the slope with respect to space.
D’Alambert’s Wave Equation

The wave equation, and modifications of it, are also found in elasticity, quantum mechanics, plasma physics and general relativity.

Scalar wave equation in one space dimension

The wave equation in one space dimension can be written as follows:

\[
\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}
\]

This equation is typically described as having only one space dimension "x", because the only other independent variable is the time "t". Nevertheless, the dependent variable "u" may represent a second space dimension, if, for example, the displacement "u" takes place in y-direction, as in the case of a string that is located in the x-y plane.

Derivation of the wave equation

The wave equation in one space dimension can be derived in a variety of different physical settings. Most famously, it can be derived for the case of a string that is vibrating in a two-dimensional plane, with each of its elements being pulled in opposite directions by the force of tension.\[^{6}\]

Another physical setting for derivation of the wave equation in one space dimension utilizes Hooke's Law. In the theory of elasticity, Hooke's Law is an

French scientist Jean-Baptiste le Rond d'Alembert (b. 1717) discovered the wave equation in one space dimension.\[^{5}\]
L’Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers (Diderot and D’Alembert)
F1 air vibration again
Perturbation theory for 1-mass-spring: \( f = \sqrt{\frac{k}{m}} \)

You need two physical elements for vibration to occur:

1. Mass \( m \): When a force is applied the mass keeps on moving, and once a mass moves it takes force to stop it from moving.

2. Spring \( k \): When a spring is compressed or stretched, a restoring force arises to return it to its neutral length.

When the two components are connected we get a mass-spring system.

1. Left alone, there is no movement so the mass doesn’t keep on moving, and no internal restoring force in the spring, so system just sits there.

2. Pull spring. Internal force is generated to get spring back to its original position. When the spring gets back to its original spring, it’s satisfied, but now the mass is moving, so it keeps on moving because there is no force to stop it. Now the mass’s movement compresses the spring, generating a force to stop it, but now the spring is compressed, so an internal force is generated, etc.

Perturbation theory for mass-spring system: if mass increases, frequency goes down, and if stiffness increases, frequency goes up.
Perturbation theory for 1-mass-spring: \( f = \sqrt{\frac{k}{m}} \)

- You need two physical elements for vibration to occur:
Perturbation theory for 1-mass-spring: \( f = \sqrt{\frac{k}{m}} \)

- You need two physical elements for vibration to occur:
  1. Mass \( m \): When a force is applied the mass keeps on moving, and once a mass moves it takes force to stop it from moving.
Perturbation theory for 1-mass-spring: \( f = \sqrt{\frac{k}{m}} \)

- You need two physical elements for vibration to occur:
  1. Mass \( m \): When a force is applied the mass keeps on moving, and once a mass moves it takes force to stop it from moving.
  2. Spring \( k \): When a spring is compressed or stretched, a restoring force arises to return it to its neutral length.
Perturbation theory for 1-mass-spring: \( f = \sqrt{\frac{k}{m}} \)

- You need two physical elements for vibration to occur:
  1. Mass \( m \): When a force is applied the mass keeps on moving, and once a mass moves it takes force to stop it from moving.
  2. Spring \( k \): When a spring is compressed or stretched, a restoring force arises to return it to its neutral length.
- When the two components are connected we get a mass-spring system.
Perturbation theory for 1-mass-spring: \( f = \sqrt{\frac{k}{m}} \)

- You need two physical elements for vibration to occur:
  1. Mass \( m \): When a force is applied the mass keeps on moving, and once a mass moves it takes force to stop it from moving.
  2. Spring \( k \): When a spring is compressed or stretched, a restoring force arises to return it to its neutral length.
- When the two components are connected we get a mass-spring system.
  1. Left alone, there is no movement so the mass doesn’t keep on moving, and no internal restoring force in the spring, so system just sits there.
Perturbation theory for 1-mass-spring: \( f = \sqrt{\frac{k}{m}} \)

- You need two physical elements for vibration to occur:
  1. Mass m: When a force is applied the mass keeps on moving, and once a mass moves it takes force to stop it from moving.
  2. Spring k: When a spring is compressed or stretched, a restoring force arises to return it to its neutral length.

- When the two components are connected we get a mass-spring system.
  1. Left alone, there is no movement so the mass doesn’t keep on moving, and no internal restoring force in the spring, so system just sits there.
  2. Pull spring. Internal force is generated to get spring back to its original position. When the spring gets back to its original spring, it’s satisfied, but now the mass is moving, so it keeps on moving because there is no force to stop it. Now the mass’s movement compresses the spring, generating a force to stop it, but now the spring is compressed, so an internal force is generated, etc.
Perturbation theory for 1-mass-spring: \( f = \sqrt{\frac{k}{m}} \)

- You need two physical elements for vibration to occur:
  1. Mass \( m \): When a force is applied the mass keeps on moving, and once a mass moves it takes force to stop it from moving.
  2. Spring \( k \): When a spring is compressed or stretched, a restoring force arises to return it to its neutral length.

- When the two components are connected we get a mass-spring system.
  1. Left alone, there is no movement so the mass doesn’t keep on moving, and no internal restoring force in the spring, so system just sits there.
  2. Pull spring. Internal force is generated to get spring back to its original position. When the spring gets back to its original spring, it’s satisfied, but now the mass is moving, so it keeps on moving because there is no force to stop it. Now the mass’s movement compresses the spring, generating a force to stop it, but now the spring is compressed, so an internal force is generated, etc.

- Perturbation theory for mass-spring system: if mass increases, frequency goes down, and if stiffness increases, frequency goes up.
Imagine you could increase the mass of a one-mass system over time, what would happen to its frequency?

Imagine you could increase the stiffness of a one-mass system over time, what would happen to its frequency?
Perturbation of mass and stiffness in time

- Imagine you could increase the mass of a one-mass system over time, what would happen to its frequency?
Perturbation of mass and stiffness in time

- Imagine you could increase the mass of a one-mass system over time, what would happen to its frequency?
- Imagine you could increase the stiffness of a one-mass system over time, what would happen to its frequency?

Spectrogram of one mass system
Multiple masses

For every mass that is added, an additional mode is found, with higher frequency. A string can be divided into an infinite number of masses, so can oscillate at an infinite number of frequencies. Very important, systems can vibrate in any number of modes at the same time!
Multiple masses

- For every mass that is added, an additional mode is found, with higher frequency.
Multiple masses

- For every mass that is added, an additional mode is found, with higher frequency.
- A string can be divided into an infinite number of masses, so can oscillate at an infinite number of frequencies
Multiple masses

- For every mass that is added, an additional mode is found, with higher frequency.
- A string can be divided into an infinite number of masses, so can oscillate at an infinite number of frequencies.
- Very important, systems can vibrate in any number of modes at the same time!
Multiple masses

- For every mass that is added, an additional mode is found, with higher frequency.
- A string can be divided into an infinite number of masses, so can oscillate at an infinite number of frequencies.
- Very important, systems can vibrate in any number of modes at the same time!
Infinite modes of string

\[ f_n = nf_1 \]

1. Fundamental
2. \( f_2 = 2f_1 \)
3. \( f_n = nf_1 \)
4.
5.
6.
Perturbation of a stiffness in a two-mass system

Two-mass system has two modes

Increase the stiffness of the middle spring in the lower frequency mode. What happens to frequency?

Increase the stiffness of the middle spring in the higher frequency mode. What happens to frequency?
Perturbation of a stiffness in a two-mass system

- Two-mass system has two modes
Perturbation of a stiffness in a two-mass system

- Two-mass system has two modes
- Increase the stiffness of the middle spring in the lower frequency mode. What happens to frequency?
- Increase the stiffness of the middle spring in the higher frequency mode. What happens to frequency?
Perturbation of a stiffness in a two-mass system

- Two-mass system has two modes
- Increase the stiffness of the middle spring in the lower frequency mode. What happens to frequency?
- Increase the stiffness of the middle spring in the higher frequency mode. What happens to frequency?
Perturbation of a mass in a three-mass system

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
<td>3.5</td>
<td>Time</td>
<td>Frequency (Hz)</td>
<td></td>
</tr>
</tbody>
</table>

Increase the mass of the middle mass. What happens to frequency?

First Mode's frequency drops, second mode stays the same.
Perturbation of a mass in a three-mass system

- Three-mass system has three modes

Increase the mass of the middle mass. What happens to frequency?

- First Mode's frequency drops, second mode stays the same
Perturbation of a mass in a three-mass system

- Three-mass system has three modes
- Increase the mass of the middle mass. What happens to frequency?
Perturbation of a mass in a three-mass system

- Three-mass system has three modes
- Increase the mass of the middle mass. What happens to frequency?
- First Mode’s frequency drops, second mode stays the same
Vibration of air in vocal tract:

http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

Air has stiffness and mass, so it has conditions for oscillating dynamical system. A constriction increases mass and increases stiffness! The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end. This is like a chain of masses and springs fixed to the wall on one side with a spring, and at the other end has a mass that's free to move. There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end. At the open end, there can be lots of movement of the air, but pressure can't build up. This chain will have many modes, with higher and higher frequencies. What will happen if we increase the mass at the free end? A mass at the free end is a constriction at the free end, a labial constriction. Therefore: Labial constrictions lower all formants! Smiley Face
Air has stiffness and mass, so it has conditions for oscillating dynamical system.
Vibration of air in vocal tract:
http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
Vibration of air in vocal tract:

http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
- The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end.
Vibration of air in vocal tract:
http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
- The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end.
- This is like a chain of masses and springs fixed to the wall on one side with a spring, and at the other end has a mass that’s free to move.
Vibration of air in vocal tract:
http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
- The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end.
- This is like a chain of masses and springs fixed to the wall on one side with a spring, and at the other end has a mass that’s free to move.
- There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end.
Vibration of air in vocal tract:
http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
- The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end.
- This is like a chain of masses and springs fixed to the wall on one side with a spring, and at the other end has a mass that’s free to move.
- There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end.
- At the open end, there can be lots of movement of the air, but pressure can’t build up.
Vibration of air in vocal tract:
http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
- The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end.
- This is like a chain of masses and springs fixed to the wall on one side with a spring, and at the other end has a mass that’s free to move.
- There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end.
- At the open end, there can be lots of movement of the air, but pressure can’t build up.
- This chain will have many modes, with higher and higher frequencies.
Vibration of air in vocal tract:
http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
- The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end.
- This is like a chain of masses and springs fixed to the wall on one side with a spring, and at the other end has a mass that’s free to move.
- There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end.
- At the open end, there can be lots of movement of the air, but pressure can’t build up.
- This chain will have many modes, with higher and higher frequencies.
- What will happen if we increase the mass at the free end?
Vibration of air in vocal tract:
http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
- The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end.
- This is like a chain of masses and springs fixed to the wall on one side with a spring, and at the other end has a mass that’s free to move.
- There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end.
- At the open end, there can be lots of movement of the air, but pressure can’t build up.
- This chain will have many modes, with higher and higher frequencies.
- What will happen if we increase the mass at the free end?

Therefore: Labial constrictions lower all formants!
Vibration of air in vocal tract:
http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

- Air has stiffness and mass, so it has conditions for oscillating dynamical system.
- A constriction increases mass and increases stiffness!
- The vocal tract can be thought of as a tube closed at the glottal end, but open at the lip end.
- This is like a chain of masses and springs fixed to the wall on one side with a spring, and at the other end has a mass that’s free to move.
- There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end.
- At the open end, there can be lots of movement of the air, but pressure can’t build up.
- This chain will have many modes, with higher and higher frequencies.
- What will happen if we increase the mass at the free end?

Khalil Iskarous
University of Southern California
Articulatory-Acoustic Relations: From (very) basic principles to simulation
Paris Diderot
November 24, 2015
17 / 89
Vibration of air in vocal tract

Remember: There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end. At the open end, there can be lots of movement of the air, but pressure can’t build up.

So in the back of the vocal tract/mass-spring chain, stiffness dominates, and in the front, motion dominates.

Now imagine that we add a constriction one third from the lips? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?

Now imagine that we add a constriction one third from the glottis? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?

So: /i/ has a low F1 and /a/ has a high F1.
Remember: There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end. At the open end, there can be lots of movement of the air, but pressure can’t build up.
Vibration of air in vocal tract

- Remember: There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end. At the open end, there can be lots of movement of the air, but pressure can't build up.

- So in the back of the vocal tract/mass-spring chain, stiffness dominates, and in the front, motion dominates.

- Now imagine that we add a constriction one third from the lips? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?

- Now imagine that we add a constriction one third from the glottis? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?

- So: /i/ has a low F1 and /a/ has a high F1.
Vibration of air in vocal tract

- Remember: There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end. At the open end, there can be lots of movement of the air, but pressure can’t build up.
- So in the back of the vocal tract/mass-spring chain, stiffness dominates, and in the front, motion dominates.
- Now imagine that we add a constriction one third from the lips? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?

So: /i/ has a low F1 and /a/ has a high F1.
Vibration of air in vocal tract

- Remember: There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end. At the open end, there can be lots of movement of the air, but pressure can’t build up.
- So in the back of the vocal tract/mass-spring chain, stiffness dominates, and in the front, motion dominates.
- Now imagine that we add a constriction one third from the lips? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?
- Now imagine that we add a constriction one third from the glottis? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?
Vibration of air in vocal tract

- Remember: There is little movement of air in the closed end, but pressure can build up, i.e. stiffness dominates at the closed end. At the open end, there can be lots of movement of the air, but pressure can’t build up.

- So in the back of the vocal tract/mass-spring chain, stiffness dominates, and in the front, motion dominates.

- Now imagine that we add a constriction one third from the lips? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?

- Now imagine that we add a constriction one third from the glottis? Would we be affecting the mass or the stiffness? Would we therefore raise or lower the frequency of vibration?

- So: /i/ has a low F1 and /a/ has a high F1.
Sensitivity Functions

F1 Sensitivity Function

F2 Sensitivity Function

F3 Sensitivity Function
Assume we have some system, and some state $x$ of the system. Here's a simple dynamical system, a rule relating the current state to the change in the state.

\[
\text{change in } x \text{ wrt time} = -.33 \times x
\]
A simple dynamical system

change in $x = -0.33 \times x$
A simple dynamical system

change in $x = -0.33 \times x$
A simple dynamical system

change in $x = -0.33 \times x$
A simple dynamical system

change in $x = -0.33 \times$
A simple dynamical system

change in $x = -0.33 \times$
A simple dynamical system

change in $x = -0.33 \times$
A simple dynamical system

\[ \text{change in } x = -.33 \times \]
A simple dynamical system

change in $x = -0.33 \times x$
A simple dynamical system

change in $x = -0.33 \times x$
A simple dynamical system

change in $x = -0.33 \times$
A simple dynamical system

dechange in $x = -0.33 \times x$
A simple dynamical system

change in $x = -0.33 \times$
A simple dynamical system

change in $x = -0.33 \times x$
A simple dynamical system

change in $x = -0.33 \times$
A simple dynamical system

\[ \text{change in } x = -0.33 \times \]
A simple dynamical system

\[ \text{change in } x = -0.33 \times x \]
change in $x = -.33 \times$
A simple dynamical system

change in $x = -0.33 \times$
A simple dynamical system

change in $x = -0.33 \times x$

Claim: This very simple dynamical system has a goal, and its goal is 0.
A simple dynamical system, Goal = 33

\[ \text{change in } x = -0.33 \times + 10 \]
A simple dynamical system, Goal = 33

\[ \text{change in } x = -0.33 \times + 10 \]
A simple dynamical system, \( \text{Goal} = 33 \)

change in \( x = -0.33 \times + 10 \)
A simple dynamical system, Goal = 33

change in $x = -0.33 \times + 10$
A simple dynamical system, Goal = 33

change in $x = -.33 \times + 10$
A simple dynamical system, Goal $= 33$

change in $x = -0.33 \times + 10$
A simple dynamical system, Goal = 33

change in $x = -0.33 \times + 10$
A simple dynamical system, Goal = 33

change in $x = -0.33x + 10$
A simple dynamical system, Goal = 33

change in $x = -0.33x + 10$
A simple dynamical system, Goal = 33

change in $x = -0.33x + 10$
A simple dynamical system, \( \text{Goal} = 33 \)

\[
\text{change in } x = -0.33 \times + 10
\]
A simple dynamical system, Goal = 33

change in $x = -0.33 \times + 10$
A simple dynamical system, \( \text{Goal} = 33 \)

change in \( x = -0.33 \times + 10 \)
A simple dynamical system, Goal = 33

change in $x = -0.33x + 10$
A simple dynamical system, Goal = 33

change in $x = -0.33 \times + 10$
A simple dynamical system, $\text{Goal} = 33$

change in $x = -0.33 \times 10$
A simple dynamical system, Goal = 33

change in $x = -.33 \times + 10$
A simple dynamical system, $\text{Goal} = 33$

change in $x = -0.33 \times + 10$
A simple dynamical system, Goal = 33

change in $x = -0.33 \times 10$
A simple dynamical system, Goal = 33

change in $x = -0.33x + 10$ Claim: This very simple dynamical system has a goal, and its goal is 33.
A simple dynamical system, Goal $= -28$, achieved Fast change in $x = -0.7x - 20$
A simple dynamical system, Goal = -28, achieved Fast change in $x = -.7 \times -20$
A simple dynamical system, \( \text{Goal} = -28 \), achieved fast change in \( x = -0.7 \times -20 \).
A simple dynamical system, Goal = -28, achieved Fast change in $x = -0.7 \times -20$
A simple dynamical system, Goal = -28, achieved Fast change in \( x = -0.7 \times -20 \)
A simple dynamical system, Goal $= -28$, achieved Fast change in $x = -0.7 \times -20$
A simple dynamical system, Goal = -28, achieved Fast change in $x = -0.7 \times -20$.
A simple dynamical system, \( \text{Goal} = -28 \), achieved Fast change in \( x = -0.7 \times -20 \)
A simple dynamical system, Goal = -28, achieved Fast change in $x = -.7 \times -20$
A simple dynamical system, \( \text{Goal} = -28 \), achieved fast change in \( x = -0.7 \times -20 \)
A simple dynamical system, Goal = -28, achieved Fast change in x = -.7 x - 20
A simple dynamical system, \( \text{Goal} = -28 \), achieved Fast change in \( x = -0.7 \times -20 \)
A simple dynamical system, Goal = -28, achieved Fast change in $x = -0.7 \times -20$
A simple dynamical system, Goal = -28, achieved Fast change in x = -.7 x - 20
A simple dynamical system, Goal = -28, achieved Fast change in $x = -.7 \times -20$
A simple dynamical system, $\text{Goal} = -28$, achieved Fast change in $x = -.7 \times -20$
A simple dynamical system, Goal = -28, achieved Fast change in \( x = -.7 \times -20 \)
A simple dynamical system, Goal = -28, achieved Fast change in $x = -0.7 \times -20$
A simple dynamical system, Goal = -28, achieved Fast change in \( x = -0.7 \times -20 \)
A simple dynamical system, Goal = -28, achieved Fast change in $x = -.7 \times -20$
The math is easy, but what has been done with such simple math is easy to miss.

The dynamical system \( \frac{dx}{dt} = kx + C \), with \( k \) from 0 to -1 seeks a goal \( C_k \) at an overall pace \( k \), slow for \( k \) near 0, and very fast for \( k \) near -1.
The math is easy, but what has been done with such simple math is easy to miss.
The math is easy, but what has been done with such simple math is easy to miss.

The dynamical system $\frac{dx}{dt} = kx + C$, with $k$ from 0 to -1 seeks a goal $\frac{C}{k}$ at an overall pace $k$, slow for $k$ near 0, and very fast for $k$ near -1.
Rate of Goal Attainment

- The closer the value of $k$ to -1, the faster the goal is attained.
Example: $\frac{dx}{dt} = -0.333x + 830$ (cf. Lindblom, 1963)
Moving to /d/ from additional vowels

You don't have to learn how to produce a different movement for every context in which /d/ occurs, just set the dynamical system
Moving to /d/ from additional vowels

- You don’t have to learn how to produce a different movement for every context in which /d/ occurs, just set the dynamical system
What if the \( k \) is positive?

Change in \( x = 0.5 \times \), starting at 100 to -100
Feedback view of simple dynamical system, Goal = 33

change in $x = -.33 \times + 10$ can also be written as: change in $x = -.33 (x - 33)$ This is an error correction, negative feedback system.
Second Interaction: More Variables in the same system

Now imagine that at the same unit, we have some other variable $y$.

One possibility is that the complete dynamical system is, for instance:

\[
\begin{align*}
\frac{dx}{dt} &= x - x^2, \\
\frac{dy}{dt} &= -4y.
\end{align*}
\]

We say: the variables don't interact.

What if we have:

\[
\begin{align*}
\frac{dx}{dt} &= 5x + 3y, \\
\frac{dy}{dt} &= -4y.
\end{align*}
\]

Here, $y$ assists $x$, but $x$ does not influence $y$.

Note that we now have a mathematical definition of within-system interaction.
Now imagine that at the same unit, we have some other variable $y$. We say: the variables don’t interact. What if we have:

$$\frac{dx}{dt} = x - x^2,$$
$$\frac{dy}{dt} = -4y.$$

$y$ assists $x$, but $x$ does not influence $y$. Note that we now have a mathematical definition of within-system interaction.
Now imagine that at the same unit, we have some other variable $y$. One possibility is that the complete dynamical system is, for instance:
\[
\frac{dx}{dt} = x - x^2, \quad \frac{dy}{dt} = -.4y
\]

$y$ assists $x$, but $x$ does not influence $y$. Note that we now have a mathematical definition of within-system interaction.
Now imagine that at the same unit, we have some other variable $y$.

One possibility is that the complete dynamical system is, for instance:

\[
\frac{dx}{dt} = x - x^2, \quad \frac{dy}{dt} = -0.4y
\]

We say: the variables don't interact.
Now imagine that at the same unit, we have some other variable \( y \).

One possibility is that the complete dynamical system is, for instance:

\[
\frac{dx}{dt} = x - x^2, \quad \frac{dy}{dt} = -0.4y
\]

We say: the variables don’t interact.

What if we have:

\[
\frac{dx}{dt} = 0.5x + 0.3y, \quad \frac{dy}{dt} = -0.4y
\]
Now imagine that at the same unit, we have some other variable $y$. One possibility is that the complete dynamical system is, for instance:

$$\frac{dx}{dt} = x - x^2, \quad \frac{dy}{dt} = -0.4y$$

We say: the variables don’t interact.

What if we have:

$$\frac{dx}{dt} = 0.5x + 0.3y, \quad \frac{dy}{dt} = -0.4y$$

$y$ assists $x$, but $x$ does not influence $y$. 
Second Interaction: More Variables in the same system

- Now imagine that at the same unit, we have some other variable \( y \).
- One possibility is that the complete dynamical system is, for instance:
  \[
  \frac{dx}{dt} = x - x^2, \quad \frac{dy}{dt} = -.4y
  \]
- We say: the variables don’t interact.
- What if we have:
  \[
  \frac{dx}{dt} = .5x + .3y, \quad \frac{dy}{dt} = -.4y
  \]
- \( y \) assists \( x \), but \( x \) does not influence \( y \).
- Note that we now have a mathematical definition of within-system interaction.
Closed Systems

What if we have:
\[ \frac{dx}{dt} = -3y, \quad \frac{dy}{dt} = 3x \]
x helps y, which in turn destroys x, or

What if we have:
\[ \frac{dx}{dt} = 3y, \quad \frac{dy}{dt} = -3x \]
x uses y, and y nourishes x

This system doesn't interact with the environment. The only interaction is between the variables within the system.

Now we can use the terms: excitation, inhibition, activator, inhibitor.
Closed Systems

- What if we have: \( \frac{dx}{dt} = -0.3y \), \( \frac{dy}{dt} = 0.3x \)

- This system doesn't interact with the environment. The only interaction is between the variables within the system.

- Now we can use the terms: excitation, inhibition, activator, inhibitor.
Closed Systems

- What if we have: \( \frac{dx}{dt} = -0.3y, \frac{dy}{dt} = 0.3x \)
- \( x \) helps \( y \), which in turn destroys \( x \), or
Closed Systems

- What if we have: $\frac{dx}{dt} = -.3y$, $\frac{dy}{dt} = .3x$
- $x$ helps $y$, which in turn destroys $x$, or
- What if we have: $\frac{dx}{dt} = .3y$, $\frac{dy}{dt} = -.3x$
Closed Systems

- What if we have: \( \frac{dx}{dt} = -0.3y, \frac{dy}{dt} = 0.3x \)
- \( x \) helps \( y \), which in turn destroys \( x \), or
- What if we have: \( \frac{dx}{dt} = 0.3y, \frac{dy}{dt} = -0.3x \)
- \( x \) uses \( y \), and \( y \) nourishes \( x \)
Closed Systems

- What if we have: \( \frac{dx}{dt} = -.3y, \frac{dy}{dt} = .3x \)
- \( x \) helps \( y \), which in turn destroys \( x \), or
- What if we have: \( \frac{dx}{dt} = .3y, \frac{dy}{dt} = -.3x \)
- \( x \) uses \( y \), and \( y \) nourishes \( x \)
- This system doesn’t interact with the environment. The only interaction is between the variables within the system.
Closed Systems

- What if we have: $\frac{dx}{dt} = -0.3y$, $\frac{dy}{dt} = 0.3x$
- $x$ helps $y$, which in turn destroys $x$, or
- What if we have: $\frac{dx}{dt} = 0.3y$, $\frac{dy}{dt} = -0.3x$
- $x$ uses $y$, and $y$ nourishes $x$
- This system doesn’t interact with the environment. The only interaction is between the variables within the system.
- Now we can use the terms: excitation, inhibition, activator, inhibitor.
These are closed systems: if $x$ increases, $y$ decreases, and if $y$ increases, $x$ decreases. Something is conserved in the system, and there’s a tradeoff, a balance.

The phase relation between the variables is born of their interaction.

Example: One of the most fundamental microcomputations in animal life: the action potential.
These are closed systems: If $x$ increases, $y$ decreases, and if $y$ increases, $x$ decreases. Something is conserved in the system, and there’s a tradeoff, a balance.
These are closed systems: If $x$ increases, $y$ decreases, and if $y$ increases, $x$ decreases. Something is conserved in the system, and there’s a tradeoff, a balance.

The phase relation between the variables is born of their interaction.
These are closed systems: If $x$ increases, $y$ decreases, and if $y$ increases, $x$ decreases. Something is conserved in the system, and there’s a tradeoff, a balance.

*The phase relation between the variables is born of their interaction.*

Example: One of the most fundamental micro-computations in animal life: the action potential.
Example: Harmonic Oscillator, the quintessential Hamiltonian System

Imagine a mass attached to a spring, sitting on a smooth table, with spring attached to the wall. The position of the mass when the spring is in neutral position is 0. Pull the mass and hold it: $x = 5$, let’s say, and $v = 0$. Let the mass go. $x$ will drop towards 0, but $v$ will rise to its maximum when $x = 0$. $x$ will no become negative, and $v$ will reduce.

\[ \frac{dx}{dt} = v, \quad \frac{dv}{dt} = -kx, \text{ i.e. } ma + kx = 0 \]